

Design and Selection of Programming Languages

18 October 2006

Exercise 6.1 — Haskell Evaluation (25% of 90 minutes Midterm 2, 2005)

Let the following Haskell definition be given:

```
from k = k : from (k+1)

prune True xs = []
prune False xs = xs

eat p [] = from (7 * 8)
eat p (x : xs) = x : prune (p x) (eat (not . p) xs)
```

Simulate Haskell evaluation for the following expression, i.e., write down **the complete sequence of intermediate expressions**:

```
eat (< 5) (from 5)
```

Note: You may introduce *abbreviations for repeated subexpressions*, or use *repetition marks for material that is unchanged from the previous line*.

Exercise 6.2 — Haskell Typing (22% of Midterm 2, 2005)

Provide **detailed derivations** of the **most general** Haskell types of the following functions:

```
maybe x f Nothing = x
maybe x f (Just y) = f y

keepof2 k h (x,y) = k (curry h x) y
```

Remember: $\text{curry} :: ((a, b) \rightarrow c) \rightarrow a \rightarrow b \rightarrow c$

Exercise 6.3 — Defining Haskell Functions (19% of Midterm 2, 2005)

Define the following Haskell functions (the solutions are independent of each other, but each can use functions specified in previous items):

(a) $\approx 5\%$ $\text{inits} :: [a] \rightarrow [[a]]$

such that $\text{inits } xs$ evaluates to a list consisting of exactly all prefixes of xs (in which order is irrelevant).

E.g., `inits [1,2,3] = [[]],[1],[1,2],[1,2,3]`

(This is a function exported by the standard library module `List`.)

(b) `fromThen :: Integer → Integer → [Integer]`

such that `fromThen x1 x2 = [x1, x2 ..]`.

(c) `fromThenTo :: Integer → Integer → Integer → [Integer]`

such that `fromThenTo x1 x2 x3 = [x1, x2 .. x3]`, e.g.:

`fromThenTo 5 7 9 = [5,7,9]`

`fromThenTo 5 7 10 = [5,7,9]`

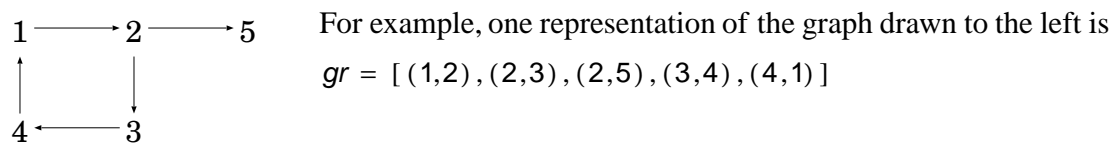
`fromThenTo 7 5 10 = []`

`fromThenTo 7 5 1 = [7,5,3,1]`

Note: `fromThen` and `fromThenTo` are the functions underlying the syntactic sugar `[1, 3 ..]` and `[1,3 .. 10]` — you should not use this syntax to define these functions.

Exercise 6.4 — Simple Graphs (34% of Midterm 2, 2005)

A simple graph can be (naïvely) represented in Haskell as a list of pairs, where an edge from node x to node y is represented by the pair (x, y) , and the sequencing of pairs in the list does not matter.



Let the following type synonym be given:

type `Graph a = [(a, a)]`

(a) `successors :: Eq a ⇒ Graph a → a → [a]` such that `successors g n` returns a list containing exactly the endnodes of those edges of the graph g that start at node n .

E.g., `successors gr 2 = [3, 5]` and `successors gr 5 = []`

(b) `pathGraph :: [a] → Graph a`

such that `pathGraph [x1, ..., xn]` evaluates to the list `[(x1, x2), ..., (xn-1, xn)]` containing the pairs of immediately consecutive elements in `xs`, e.g.,

`pathGraph [2,3,4,1,2,5] = [(2,3), (3,4), (4,1), (1,2), (2,5)]`, which is just another representation for the graph drawn above.

(c) `hasCycle :: Eq a ⇒ [a] → Bool` A *path* in a simple graph can be represented as a list of nodes, as above in (b). Define the Haskell function `hasCycle p` such that `hasCycle p` is true if path p contains a cycle, i.e., if there is a node that occurs at least twice in p . For example, the path `[2,3,4,1,2,5]` has a cycle around node 2.

(d) `edgeGraph :: Eq a ⇒ Graph a → Graph (a, a)` such that `edgeGraph g` returns the *edge graph* of g . This edge graph has edges of g as nodes, and has an edge from $e1$ to $e2$ iff the end node of $e1$ is equal to the start node of $e2$ (as edges in g).

(e) `paths :: Eq a ⇒ Graph a → [[a]]` Define `paths` to calculate all non-empty cycle-free paths of a graph.